# Linear Algebra – Day 28

MATH 220

**1. Milo:** I wonder what it means for $\lambda = 0$ to be an eigenvalue of $A$.

Read this aloud at your table!

**Delphine:** Milo, what it means for $\lambda$ to be an eigenvalue is written on the screen!

**Milo:** Oh, right! So let's read what's on the screen and pretend $\lambda = 0$.

**Delphine:** Wow, something amazing happened!

**Group Chat:** What amazing thing happens if $\lambda = 0$ is an eigenvalue of $A$?

**2.** Consider the upper-triangular matrices $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

(a) What are the roots, with multiplicities, of $\det(M - \lambda I)$? How about for $\det(N - \lambda I)$?

(b) For matrix $M$, what are $\dim(E_3)$ and $\dim(E_4)$?

(c) For matrix $N$, what are $\dim(E_3)$ and $\dim(E_4)$?

STOP *and wait for further instructions*

(d) For $M$: $\text{alg}(3) =$ ____ $\text{geo}(3) =$ ____ $\text{alg}(4) =$ ____ $\text{geo}(4) =$ ____

(e) For $N$: $\text{alg}(3) =$ ____ $\text{geo}(3) =$ ____ $\text{alg}(4) =$ ____ $\text{geo}(4) =$ ____

**3. Maura:** Wow! This means that if $\lambda$ only occurs once as a root of the characteristic polynomial of $A$, we *already* know that $\lambda$ will produce *only one* dimension worth of eigenvectors. We would not even have to calculate the eigenvectors!

**Group Chat:** What is Maura talking about?

**4.** (a) Write down any $2 \times 2$ diagonal matrix $A$ and any $3 \times 3$ diagonal matrix $B$.

☞ To make it interesting, use different diagonal entries, none equal to 0 or 1.

(b) Find $A^2$ and $B^2$.

(c) Find $A^3$ and $B^3$.

(d) Do you have a guess as to what $A^k$ and $B^k$ equal?

(e) If $D = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_n \end{bmatrix}$, then what is $D^k$?

(f) **Theorem:** If $D$ is the previous diagonal matrix, then the eigenvalues of $D^k$ are

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**5.** Suppose $A$ is an $n \times n$ matrix and $A = PBP^{-1}$ for some $n \times n$ matrices $P$ and $B$.

(a) Find $A^2$ and $A^3$ in terms of $P$ and $B$.

(b) What do you think $A^k$ equals (in terms of $P$ and $B$)?

**6.** **Ava:** Don't ask me why, but I really want to know $A^{63}$ for $A = \begin{bmatrix} 1 & 3 & -3 & -3 \\ 3 & -3 & 3 & -1 \\ -3 & 3 & 1 & -3 \\ -3 & -1 & -3 & -3 \end{bmatrix}$.

**Jonah:** Huh? Why 63? OK, well you can easily find $A^{63}$ using this information:

$$\begin{bmatrix} 1 & 3 & -3 & -3 \\ 3 & -3 & 3 & -1 \\ -3 & 3 & 1 & -3 \\ -3 & -1 & -3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix}^{-1}$$

**Group chat:** Why does Jonah think Ava can calculate $A^{63}$ easily?

☞ Hint: Jonah's information looks like $A = PBP^{-1}$.