# Linear Algebra – Day 27

MATH 220

**1.** (a) Find the characteristic polynomial of $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$.

(b) What are the eigenvalues of $A$?

(c) For one of the $\lambda$ you found in part (a), find $E_\lambda$. (Write it as a span.)

**2.** Consider the matrix $C = \begin{bmatrix} 5 & 17 & \pi & 312 \\ 0 & -2 & -74 & 91 \\ 0 & 0 & 3 & 803 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

(a) QUICK! Find the characteristic polynomial of $C$.

Don't panic. Just write $C - \lambda I$ and go from there.

(b) When Jason looked at matrix $C$, he *immediately* said

**"Boom! The eigenvalues of this matrix are 5, −2, and 3."**

How did Jason know this so quickly?

(c) **Theorem:** If $A$ is a triangular matrix (*either* upper or lower), then you can find the eigenvalues of $A$ by _____.

**3.** (a) Suppose you know that $A$ is an invertible $n \times n$ matrix and $A\mathbf{x} = 3\mathbf{x}$. What is $A^{-1}\mathbf{x}$?

HINT: If $A^{-1}$ undoes $A$ and $A$ triples $\mathbf{x}$, what should $A^{-1}$ do to $\mathbf{x}$?

(b) Suppose $A$ is a $3 \times 3$ matrix whose eigenvalues are $1, 2, 3$. What are the eigenvalues of $A^{-1}$?

(c) **Conjecture**: If $A$ is an $n \times n$ invertible matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$, then the eigenvalues of $A^{-1}$ are _____.

(d) **Conjecture**: $A$ is invertible if and only if _____.

Say something about its eigenvalues.

**4.** (a) Find the characteristic polynomial and eigenvalues of $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

(b) What are the eigenvectors (in $\mathbb{R}^2$) for $A$?

(c) Recall that $A$ is the matrix that rotates vectors in $\mathbb{R}^2$ counterclockwise by 90 degrees ($\pi/2$ radians). Given that, why do the answers to (a) and (b) make sense?

(d) Use the fact that $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is reflection in the line $y = x$ to find the eigenvalues of $A$ without calculation.

Use geometry! What vectors remain multiples of themselves? What multiple?

**5.** (a) What is the characteristic polynomial of $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$? What are the eigenvalues?

(b) What are the dimensions of the eigenspaces of $M$?

(c) What is the characteristic polynomial of $N = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$? What are the eigenvalues?

(d) What are the dimensions of the eigenspaces of $N$?

(e) How are $M$ and $N$ the same? How are they different? How are their eigenvalues the same? How are their eignespaces different?