Linear Algebra – Day 26

MATH 220

1. Cleo: We have seen this situation before!

Erez: Hmm...I think I would remember the word "eigenvector." We've never seen these.

Cleo: Think, Erez! We did not *call* them eigenvectors, but we have seen them already.

Erez: When? We've never worried about the equation $A\mathbf{x} = \lambda\mathbf{x}$.

Cleo (sighing loudly): But we did this in Chapter 3, for $\lambda = 1$.

Group chat: What "old" situation is Cleo trying to remind you of?

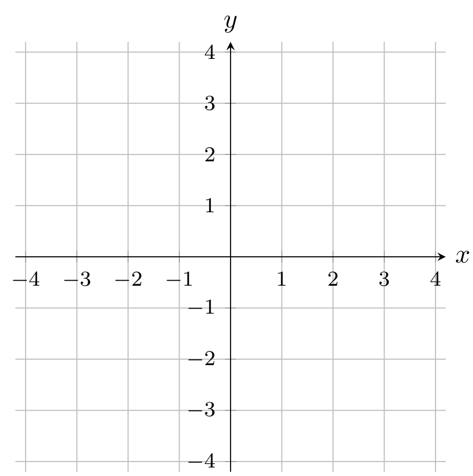
2. Let $A = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(a) **Split the work among your group members:** Compute $A\mathbf{u}$, $A\mathbf{v}$, and $A\mathbf{w}$.

(b) **Group chat:** Do any of $\mathbf{u}, \mathbf{v}$, and/or $\mathbf{w}$ satisfy the definition of what it means to be an eigenvector of A? Why or why not?

☞ This should be easy to check. Use the definition of eigenvector.

(c) Plot vectors $\mathbf{u}$, $\mathbf{v}$, $A\mathbf{u}$, and $A\mathbf{v}$ on the following coordinate plane.



(d) In part (b), you should have concluded that $\mathbf{u}$ is an eigenvector of A and $\mathbf{v}$ is not an eigenvector of A. Explain why your plots in part (c) also tell you this.

(e) Find an eigenvector of A *other than* $\mathbf{u}$ or $\mathbf{w}$.

☞ Don't guess and check, here. Remember that multiplication by A respects scalar multiplication.

3. Delphine, Milo, and Levi are now reflecting on all of their hard work on problem 2:

☞ You should definitely read this conversation out loud at your table. Hearing written words spoken out loud can help with comprehension.

Delphine: OK, I see how this works. The vector $\mathbf{u}$ is an eigenvector because $A\mathbf{u} = (-1)\mathbf{u}$. That means that $\lambda = -1$ is an eigenvalue of A.

Milo: Indeed! Great work, Delphine. AND, the vector $\mathbf{w}$ is an eigenvector because $A\mathbf{w} = 0\mathbf{w}$. That means that $\lambda = 0$ is also an eigenvalue of A.

Delphine: That makes sense, because $A\mathbf{w}$ is definitely a multiple of $\mathbf{w}$ in that case.

Levi: Interesting...the number 0 can actually be an eigenvalue of a matrix. But wait, now I am confused. Why don't we want to let $\mathbf{0}$ be an eigenvector then? If we calculate $A\mathbf{0}$ we get $\mathbf{0}$, which is definitely a multiple of $\mathbf{0}$.

Group Chat: Why is it a good thing *not* to think of $\mathbf{0}$ as an eigenvector?

☞ Hint: eigenvalue?

4. **Simon:** Hey! Did you know $\lambda = 3$ is an eigenvalue of $A = \begin{bmatrix} 4 & -2 & 5 \\ 1 & 1 & 5 \\ 1 & -2 & 8 \end{bmatrix}$?

Maura: How did you figure that out?

Simon: I have my ways. You'll have to trust me that I am correct.

Maura: I guess we should see if we can find an eigenvector that pairs with this eigenvalue.

Group chat: What does it *mean* to know that $\lambda = 3$ is an eigenvalue of A?

Group chat: What "equation" would you need to solve in order to find an eigenvector $\mathbf{x}$ that has eigenvalue $\lambda = 3$? Find at least one eigenvector $\mathbf{x}$!

☞ Hint: null space?

Simon: I didn't have time to check if $\lambda = 5$ is an eigenvalue of A or not. Is it?

☞ Pretend $\lambda = 5$ is an eigenvalue and try to find some eigenvectors. What happens?