Linear Algebra – Day 23

MATH 220

1. For each matrix below, find a basis for the row space, a basis for the column space, and a basis for the null space.

Use Mathematica!

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 4 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 & 6 & 2 \\ 2 & 1 & 5 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$

2. What do you observe about the dimensions of the spaces associated with each matrix above?

3. If A is an $n \times m$ matrix, what is the largest possible value for rank(A)? What is the smallest possible value for nullity(A)?

4. For any matrix A, how does rank(A) relate to rank(A^T)?

5. (a) Give an example of a 3×3 matrix whose column space is a plane through the origin in $\mathbb{R}^3$.

(b) What kind of geometric object is the null space of your matrix?

(c) What kind of geometric object is the row space of your matrix?

6. Let $A = \begin{bmatrix} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{bmatrix}$, where t is some number. What values of rank(A) are possible?