Linear Algebra – Day 21

MATH 220

1. Find examples of 3×3 matrices A, B, C, and D such that:

(a) $\text{nullity}(A) = 3$

(b) $\text{nullity}(B) = 2$

(c) $\text{nullity}(C) = 1$

(d) $\text{nullity}(D) = 0$

2. If possible, find examples of 3×4 matrices A, B, C, D, and E such that:

(a) $\text{nullity}(A) = 4$

(b) $\text{nullity}(B) = 3$

(c) $\text{nullity}(C) = 2$

(d) $\text{nullity}(D) = 1$

(e) $\text{nullity}(E) = 0$

3. Let A be an $n \times n$ matrix with column vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$, and let $T : \mathbb{R}^n \to \mathbb{R}^n$ the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$.

List as statements as you can that are equivalent to the following.

The columns of A span $\mathbb{R}^n$.

Remember that "equivalent" means all true or all false.

4. Suppose all you know about A is that it is a $m \times n$ matrix, and let T be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$.

(a) What are the possible values for the dimension of the kernel of T?

(b) What are the possible values for the dimension of the range of T?

(c) If you find out that the columns of A are linearly independent, how does that change your previous answers?

(d) If the columns of A are linearly independent, what else can you conclude about A and T?