Linear Algebra – Day 20

MATH 220

1. (a) The graph of $2x_1 - x_2 = 0$ is a line in $\mathbb{R}^2$. Sketch this line. Then find a basis for it.

(b) The graph of $2x_1 - x_2 - x_3 = 0$ is a _____ in $\mathbb{R}^3$. Try to sketch it. Then find a basis for it.

(c) The graph of $2x_1 - x_2 - x_3 + x_4 = 0$ is a _____ in $\mathbb{R}^4$. Find a basis for it. 3 vectors?

2. Let $A = \begin{bmatrix} 1 & 4 & 5 & 1 \\ 2 & 5 & 7 & 1 \\ 3 & 6 & 11 & 1 \end{bmatrix}$.

(a) Find null(A) by finding all solutions to $A\mathbf{x} = 0$.

(b) Find a basis for null(A) by finding a linearly independent set of vectors that spans null(A). 1 vector?

(c) The **nullity** of A is the dimension of null(A). What is the nullity of A?

(d) Find a basis for the range of A.

3. Find examples of 3×3 matrices A, B, C, and D such that:

(a) nullity(A) = 3

(b) nullity(B) = 2

(c) nullity(C) = 1

(d) nullity(D) = 0

4. Consider the sets of vectors $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix} \right\}$ and $S' = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} \right\}$.

(a) How can you tell without doing any work that S is not a basis for $\mathbb{R}^3$?

(b) Is S linearly independent or dependent?

(c) Construct a basis for $\mathbb{R}^3$ that includes the two vectors in S. How do you know you have a basis?

(d) How can you tell without doing any work that S' is not a basis for $\mathbb{R}^3$?

(e) Does S' span all of $\mathbb{R}^3$?

(f) Construct a basis for $\mathbb{R}^3$ that includes some of the vectors in S'. How do you know you have a basis?

5. If possible, find examples of 3×4 matrices A, B, C, D, and E such that:

(a) $\text{nullity}(A) = 4$

(b) $\text{nullity}(B) = 3$

(c) $\text{nullity}(C) = 2$

(d) $\text{nullity}(D) = 1$

(e) $\text{nullity}(E) = 0$

6. Suppose all you know about A is that it is a $m \times n$ matrix, and let T be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$.

(a) What are the possible values for the dimension of the kernel of T?

(b) What are the possible values for the dimension of the range of T?

(c) If you find out that the columns of A are linearly independent, how does that change your previous answers?