Linear Algebra – Day 19

MATH 220

1. With your group, write down three random vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ (all in $\mathbb{R}^n$).

☞ Make it interesting, but simple.

(a) How are $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ and $\text{span}(\mathbf{v}_3, \mathbf{v}_2, \mathbf{v}_1)$ related?

(b) How are $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ and $\text{span}(\mathbf{v}_1, 7\mathbf{v}_2, \mathbf{v}_3)$ related?

(c) How are $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ and $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 + 7\mathbf{v}_2)$ related?

2. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be the *rows* of a matrix A.

☞ THE ROWS. They are vectors, too.

(a) What happens to the span of the rows after the row operation $R_i \leftrightarrow R_j$?

☞ We did #1 for a reason.

(b) What happens to the span of the rows after the row operation $cR_i \rightarrow R_i$ ($c \neq 0$)?

☞ We did #1 for a reason.

(c) What happens to the span of the rows after the row operation $cR_i + R_j \rightarrow R_j$?

☞ We did #1 for a reason.

(d) Circle the one correct word:
Row operations *sometimes* *always* *never* change the span of the rows of a matrix.

3. For each statement, pick the correct symbol and discuss: $<$ $\leq$ $=$ $\geq$ $>$

(a) If you have m vectors that span $\mathbb{R}^3$, then m ___ 3.

(b) If you have m linearly independent vectors in $\mathbb{R}^3$, then m ___ 3.

(c) If you have m vectors that span $\mathbb{R}^n$, then m ___ n.

(d) If you have m linearly independent vectors in $\mathbb{R}^n$, then m ___ n.

(e) If you have m linearly independent that span $\mathbb{R}^n$, then m ___ n.

4. How many vectors will there be in a basis for $\mathbb{R}^n$?

5. For this problem:

$$M = \begin{bmatrix} 1 & 2 & 4 & 7 \\ 2 & -1 & 3 & 5 \\ 4 & -7 & 1 & 1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2 & 3.4 \\ 0 & 1 & 1 & 1.8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1 & 2 & 4 \\ 2 & -1 & -7 \\ 4 & 3 & 1 \\ 7 & 5 & 1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Let $S = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -7 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} \right)$ and $W = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 1 \\ 1 \end{bmatrix} \right)$.

- Using the "Row Method," a basis for W is:

- Using the "Column Method," a basis for W is:

- Using the "Row Method," a basis for S is:

- Using the "Column Method," a basis for S is:

(b) **Cleo:** WOW! EVERYONE! Look at $\text{rref}(N) = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. I can immediately see the number of vectors a basis of W will have!
Group chat: What is Cleo looking at?

(c) **Jonah:** AND, I can immediately see the number of vectors a basis of S will have!
Group chat: What is Jonah looking at?

(d) **Nadia: (looking quite excited):** I can ALSO immediately see the number of vectors a basis of $\text{null}(N)$ will have.
Group chat: What is Nadia looking at?