Linear Algebra – Day 14

MATH 220

1. Suppose $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & -2 & 3 \end{bmatrix}$.

Let T_A and T_B be the linear transformation given by multiplication by A and B respectively.

(a) Does $T_B \circ T_A$ even make sense?

If yes, the domain of $T_B \circ T_A$ is _____ and the codomain is _____ .

That is, is it possible to perform T_A first, immediately followed by T_B?

(b) Does $T_A \circ T_B$ even make sense?

If yes, the domain of $T_A \circ T_B$ is _____ and the codomain is _____ .

That is, is it possible to perform T_B first, immediately followed by T_A?

(c) You should have concluded that $T_A \circ T_B$ is the one that makes sense. Find the matrix that performs the transformation $T_A \circ T_B$.

Hint: what happens to the "e" vectors?

(d) Compute AB. What do you notice?

2. Felix: I need to multiply $\begin{bmatrix} 2 & 3 & 4 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ times $\begin{bmatrix} 3 & -1 & 5 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

Ava: You will get a 3×3 matrix with at least six entries that are 0.

Group chat: What is Ava talking about? Compute the matrix product to see if Ava is correct.

3. Milo: Matrix multiplication is hard!

Chloe: Sometimes it's not! Try multiplying the matrices $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

Group chat: What do you think Chloe means?

Group chat: What other examples of matrices can you think of that are *easy* to multiply?

4. Compute the matrix product:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 4 \\ 1 & 5 & 4 \end{bmatrix} =$$

Milo: You just did an elementary row operation!

Group chat: What is Milo talking about?

Chloe: Did you know that *every* elementary row operation is really matrix multiplication in disguise?

Group chat: What is Chloe talking about? Can you explain how each of the three types of elementary row operations can be performed by multiplying matrices?

5. Can you find each of the mystery matrices?

(a) $\begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 5 \\ 7 & 9 \end{bmatrix} \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$