Linear Algebra – Day 9

MATH 220

1. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ be a set of four mystery vectors in $\mathbb{R}^3$. In each case below, (i) decide whether the set is linearly independent or dependent, and (ii) decide whether the set spans all of $\mathbb{R}^3$.

(a) The matrix $\begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{u}_4\end{bmatrix}$ has RREF $\begin{bmatrix}1 & 0 & 0 & 2\\ 0 & 1 & 0 & 3\\ 0 & 0 & 1 & 4\end{bmatrix}$.

(b) The matrix $\begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{u}_4\end{bmatrix}$ has RREF $\begin{bmatrix}1 & 0 & 2 & 0\\ 0 & 1 & 3 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$.

(c) The matrix $\begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{u}_4\end{bmatrix}$ has RREF $\begin{bmatrix}1 & 0 & 2 & 3\\ 0 & 1 & 4 & 5\\ 0 & 0 & 0 & 0\end{bmatrix}$.

2. In parts (a) – (c), circle the correct option for each statement.

(a) $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{10}$ are ten vectors in $\mathbb{R}^9$.

The vectors must / cannot / might or might not span $\mathbb{R}^9$.

The vectors must / cannot / might or might not be linearly independent.

If these vectors span $\mathbb{R}^9$, then they must / cannot / might or might not be linearly independent.

(b) $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_8$ are eight vectors in $\mathbb{R}^9$.

The vectors must / cannot / might or might not span $\mathbb{R}^9$.

The vectors must / cannot / might or might not be linearly independent.

If these vectors are linearly independent, then they must / cannot / might or might not span $\mathbb{R}^9$.

(c) $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_9$ are nine vectors in $\mathbb{R}^9$.

The vectors must / cannot / might or might not span $\mathbb{R}^9$.

The vectors must / cannot / might or might not be linearly independent.

If these vectors span $\mathbb{R}^9$, then they must / cannot / might or might not be linearly independent.

3. Suppose A is a 4×4 matrix whose columns are linearly independent. Explain *why* the linear system whose augmented matrix is $\left[\begin{array}{c|c} A & \mathbf{b}\end{array}\right]$ has exactly one solution for all vectors $\mathbf{b} \in \mathbb{R}^4$.

4. Suppose A is a 4×4 matrix whose columns are linearly dependent. Explain *why* the linear system whose augmented matrix is $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$ has no solution for some vector $\mathbf{b} \in \mathbb{R}^4$.

5. In each case, determine whether $\mathbf{b}$ is in the span of the other vectors. If so, express $\mathbf{b}$ as a linear combination of the other vectors.

☞ use Mathematica!

(a) $\mathbf{a}_1 = \begin{bmatrix} 4 \\ -3 \\ 4 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 7 \\ 1 \\ -2 \end{bmatrix}$

(b) $\mathbf{a}_1 = \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ -4 \\ 1 \end{bmatrix}$

6. Suppose $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent vectors in $\mathbb{R}^n$. Let matrix A have these vectors as its columns. What is the RREF of A?

7. Suppose $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly dependent vectors in $\mathbb{R}^n$. Let matrix A have these vectors as its columns. What can you say about the RREF of A?