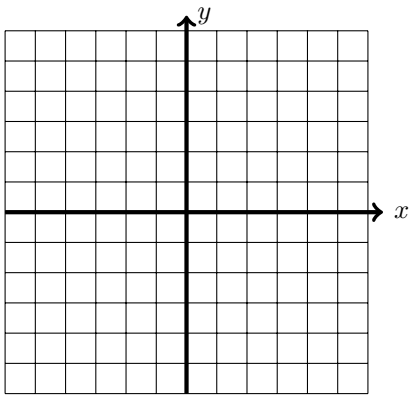
Linear Algebra – Day 5

MATH 220

1. Let $A = (2, 0)$ and $B = (6, 3)$. Draw the vector from A to B (which we will call $\mathbf{u}$).



(a) Suppose $C = (-1, -2)$ and the vector from C to D (which we will call $\mathbf{w}$) has the same length and direction as the vector $\mathbf{u}$. Find the coordinates of point D.

(b) Sketch vector $\mathbf{w}$ on the grid above. Geometrically, how are the vector $\mathbf{u}$ and the vector $\mathbf{w}$ related?

(c) Let $O = (0, 0)$ denote the origin. Find point E such that the vector from O to E *equals* the vector $\mathbf{u}$.

Unless we have a reason not to, we almost always draw vectors emanating from the origin.

2. Consider the vectors $\mathbf{u} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

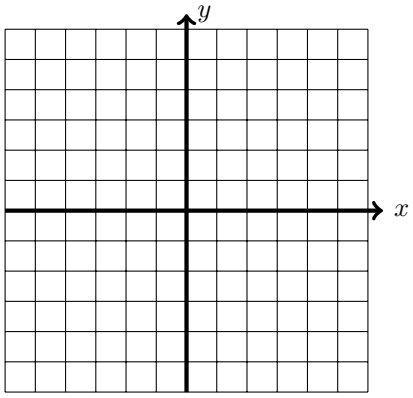
(a) What do you think $\mathbf{u} + \mathbf{v}$ should equal?

(b) What do you think $2\mathbf{v}$ should equal?

(c) Sketch $\mathbf{u}$, $\mathbf{v}$, and (your guesses for) $\mathbf{u} + \mathbf{v}$ and $2\mathbf{v}$ on the grid below (all emanating from the origin). In the sketch, how are these vectors related to each other?

(d) Use your sketch to explain why $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.

This is "obvious" algebraically. Why?



3. Is it possible to find numbers x_1, x_2, x_3 so that $x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$?

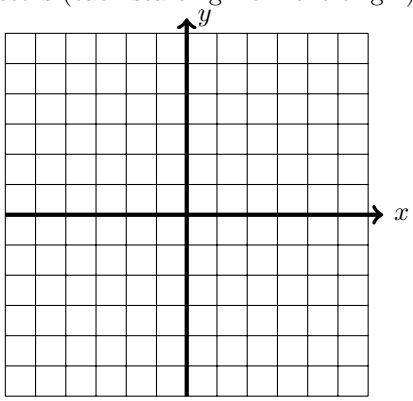
All three positions on each side of the equation must be equal.

4. (a) Can you find numbers x_1 and x_2 such that $x_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$?

(b) Can you find numbers x_1 and x_2 such that $x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$?

5. Suppose we have the vector $\mathbf{d} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(a) Draw each the following vectors (each starting from the origin): $\mathbf{d}, 2\mathbf{d}, 3\mathbf{d}$, and $\frac{5}{2}\mathbf{d}$.



(b) Suppose t is some random number that is larger than 1. What does the vector $t\mathbf{d}$ look like compared to $\mathbf{d}$?

That is, how is $t\mathbf{d}$ related to $\mathbf{d}$ when you draw them both?

(c) Draw the vector $\frac{1}{2}\mathbf{d}$ above. Suppose t is some random number between 0 and 1. What does the vector $t\mathbf{d}$ look like compared to $\mathbf{d}$?

(d) On the same grid above, draw the vectors $-\mathbf{d}, -2\mathbf{d}, -3\mathbf{d}, -\frac{5}{2}\mathbf{d}$. If t is some random negative number, describe what the vector $t\mathbf{d}$ looks like.

(e) If you look at *all* the vectors of the form $t\mathbf{d}$ together at the same time, what geometric shape do they create (if any)?