Linear Algebra – Day 3

MATH 220

1. Group chat: Which row operation are performed between each step below? Also write the augmented matrix for the system at each step.

START:

$$\begin{aligned} 2x_2 + 4x_3 &= 2 \\ x_1 + 2x_2 + 3x_3 &= 1 \\ x_2 + 2x_3 &= 1 \end{aligned}$$

STEP 1:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_2 + 4x_3 &= 2 \\ x_2 + 2x_3 &= 1 \end{aligned}$$

STEP 2:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ x_2 + 2x_3 &= 1 \\ x_2 + 2x_3 &= 1 \end{aligned}$$

STEP 3:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ x_2 + 2x_3 &= 1 \\ 0 &= 0 \end{aligned}$$

Felix: Alright Maura! The system is in echelon form. I'm ready to solve the system!

Maura: But I *LOVE* row operations! Can I just do one more, please? Here is STEP 4:

$$\begin{aligned} x_1 \qquad - x_3 &= -1 \\ x_2 + 2x_3 &= 1 \\ 0 &= 0 \end{aligned}$$

Felix: That was really useful! I find it *much* easier to solve the system now.

Group Chat: Why is it easier to solve the system after Maura's row operation?

2. Simon: I just put three different systems into an augmented matrix. Then, I did some row operations on all three. Here's what I got when I was all done.

(a) $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & 5 & 10 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & 0 & 9 \end{array}\right]$

Naila: Look, all three matrices are all in echelon form!

Group Discussion: Is Naila right? Are all three in echelon form?

Simon: This makes it easy to find the solutions to each system. Try it!

Group Discussion: What are the solutions to each system?

3. Simon: Hey Maura, I need you to to help me with some systems.

Naila: It looks like you already put each system into an augmented matrix **and** did some row operations!

Simon: Yes. I don't care what the exact solution is, but I need to know, very quickly, *how many* solutions each system has.

(a) $\left[\begin{array}{rrr|r} 1 & 1 & -1 & 2 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & 5 & 10 \end{array}\right]$

(b) $\left[\begin{array}{rrr|r} 1 & 1 & -1 & 2 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & 0 & 10 \end{array}\right]$

(c) $\left[\begin{array}{rrr|r} 1 & 1 & -1 & 2 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{rr|r} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right]$

(e) $\left[\begin{array}{rr|r} 1 & 1 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$

(f) $\left[\begin{array}{rrr|r} 1 & 1 & 4 & 0 \end{array}\right]$

4. Group Discussion/Summary: Suppose you have an augmented matrix in echelon form.

(a) How can you tell when the corresponding system is inconsistent? Explain.

☜ i.e., has 0 solutions.

(b) How can you tell when the corresponding system is consistent? Explain.

☜ i.e., has either 1 or ∞-many solutions.

(c) How can you tell when the corresponding system has *exactly* one solution? Explain.

(d) How can you tell when the corresponding system has ∞-many solutions? Explain.

5. Suppose $\star$ represents the presence of a *nonzero* number. For each of the following augmented matrices, how many solutions will the corresponding system have?

☜ HINT: Think about how back-substitution might go.

$\left[\begin{array}{ccc|c} \star & \star & \star & \star \\ 0 & \star & \star & \star \end{array}\right]$ $\left[\begin{array}{ccc|c} \star & \star & \star & \star \\ 0 & \star & \star & \star \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\left[\begin{array}{ccc|c} \star & \star & \star & \star \\ 0 & \star & \star & \star \\ 0 & 0 & 0 & \star \end{array}\right]$ $\left[\begin{array}{ccc|c} \star & \star & \star & \star \\ 0 & \star & \star & \star \\ 0 & 0 & \star & 0 \end{array}\right]$ $\left[\begin{array}{cc|c} \star & \star & \star \\ 0 & \star & \star \\ 0 & 0 & 0 \end{array}\right]$