Exam 3 Guidance

MATH 126 A/B • Fall 2025

The following is intended to help you focus your studying for Exam 3, which is on the material we've studied since Exam 2 through Wednesday, November 19. The following notes might not encompass everything you could see on the exam, but they are intended to help you think about the course content and study effectively.

Important Notes

- You may use one side of a 4×6-inch (10×15 cm) card (or a piece of paper of that side) containing notes that you prepare in advance. Anything you want can go on that card.
- Calculators will not be helpful and definitely *not necessary*. This exam will test calculus concepts, not your ability to do arithmetic. However, you may use a calculator, but only simple arithmetic, and you must state where exactly you used your calculator. You may not use a phone or computer.
- As you study, focus on fluency and deep understanding of calculus concepts rather than only pure memorization.

What should I know and be able to do?

Functions of Several Variables and 3D Space

- Understand the 3D coordinate system.
- Be able to interpret functions of two variables given by either a formula, a table of values, a 3D plot, or a contour plot.
- Understand the equations for planes and spheres in 3D space.
- Be able to compute the distance between points in 3D.
- Be able to interpret traces and contours (level curves) of functions of 2 variables.

Vectors

- Understand the idea of vectors in both 2 and 3 dimensions, and be able to compute the length (magnitude) of a vector.
- Be able to do basic operations on vectors, most importantly vector addition and scalar multiplication.
- Be able to compute the dot product of two vectors. Understand that the dot product gives you a number, and understand basic properties of dot products.
- Be able to use a dot product to decide if two vectors are orthogonal and to find angles between vectors. (You do not need to know about vector projections, though this is in Section 9.3 in

the text.)

- Be able to compute the cross product of two vectors in $\mathbb{R}^3$. Understand that the cross product gives you a vector, and understand basic properties of cross products.
- Be able to use a cross products to *find* a vector orthogonal to two given vectors.

Lines and Planes

- Understand how to find an equation for a line in parametric form, and also how to read information about a line from parametric equations for the line.
- Be able to convert an equation for a line from parametric form to vector form and vice-versa.
- Understand how to find an equation for a plane (in the form $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$) and how to read information *from* the equation for a plane (which could be expressed in either the form $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ or $ax+by+cz=d$ or something similar). You should be able to find an equation for a plane given information such as a point and a normal vector, three points in the plane, etc.

Partial Derivatives and Directional Derivatives

- Understand the meaning of partial derivatives f_x and f_y. (The rate of change in the x- or y-direction.)
- Know how to estimate partial derivatives of $f(x,y)$ when given a chart of values or a contour map.
- Know how to compute partial derivatives exactly if $f(x,y)$ is given as an equation.
- Know how to compute second partial derivatives, and know that $f_{xy}=f_{yx}$ whenever these partial derivatives are continuous.
- Understand the meaning of directional derivatives. (A rate of change in a direction $\mathbf{u}$).
- Know how to compute directional derivatives. You *must not* forget the key step of making sure the direction vector $\mathbf{u}$ is a *unit vector*. In other words, make sure that $|\mathbf{u}|=1$. If not, replace $\mathbf{u}$ with $\frac{\mathbf{u}}{|\mathbf{u}|}$ before doing anything else.
- You should be able to calculate the gradient vector $\nabla f(x,y)$ and use it to help find a directional derivative $D_{\mathbf{u}}f(x,y)$.
- You should understand that the gradient vector (i.e., $\nabla f(x,y)$) points in the direction of greatest rate of change and that the actual greatest rate of change is the *length* of the gradient vector (i.e., $|\nabla f(x,y)|$).
- You should understand that the negative of gradient vector (i.e., $-\nabla f(x,y)$) points in the direction of least rate of change and that the actual least rate of change is the *negative of the length* of the gradient vector (i.e., $-|\nabla f(x,y)|$).

- You should understand that the gradient vector at a point always points orthogonal to the level curve (contour) through that point.

Linearization

- Know how to find an equation of the tangent plane to the graph of $f(x, y)$ at a given point (a, b).
- Know how to use this equation to estimate the value of $f(x, y)$ near the point of tangency.

Common mistakes to watch out for

- Don't forget that a dot product gives you a number, while a cross product gives you a vector.
- When computing partial derivatives, make sure to treat all variables except the one being differentiated as constants. Forgetting this leads to errors, especially when applying the product rule or the chain rule.
- Small algebra mistakes happen. We'll learn from them but move on. Be careful not to make *fundamental* algebra mistakes. Square roots and other powers can't be distributed on addition, and neither can sine, cosine, or logarithms. For example:

$$\begin{aligned}
\sqrt{x^2+9} &\quad \text{IS NOT} \quad x+3 \\
(x+4)^3 &\quad \text{IS NOT} \quad x^3+4^3 \\
\frac{a}{b+c} &\quad \text{IS NOT} \quad \frac{a}{b}+\frac{a}{c} \\
\ln(a+b) &\quad \text{IS NOT} \quad \ln a+\ln b
\end{aligned}$$

How should I study?

First, understand that people learn differently and process information in different ways and at different speeds. I suggest:

- Read through each section of *Active Calculus* again and think about whether or not the main ideas make intuitive sense. Can you explain them out loud?
- The problems in each section of *Active Calculus* are great practice problems. There is also a large set of practice problems available on Edfinity.
- Do a few problems each day, ramping up as we get closer to the exam day. Talk with your classmates about the problems. Talk with me and visit the help sessions if you want to make sure things are correct or if you want to chat.
- Work on fluency! The exam is timed and you want to be able to do some of the problems efficiently. Perhaps give each other a few selected problems from a few different sections and time yourselves. This way, you won't know which section the problem came from.
- COME VISIT PROF. WRIGHT AND ASK QUESTIONS.