Exam 3 Practice Problems

1. Let $f(x, y) = e^{-x^2-y^2}$.

(a) Make a guess for what the graph of this function looks like without using technology. (*Hint*: What is $f(0, 0)$? What happens to $f(x, y)$ as (x, y) move away from the origin?)

(b) Use technology to draw the graph. Was your guess correct?

(c) Sketch the contour diagram for this function.

2. Explain what is wrong with the following statement: "Since $x^2 + y^2 = 1$ is the equation of a circle, the graph of $f(x, y) = x^2 + y^2$ is a circle."

3. Is it true that $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$? Either explain why this is true for all vectors, or give a counterexample that shows it is not true for some vectors.

4. Find the equation of the plane through the points $(2, 0, 3)$, $(5, 2, -2)$, and $(1, 4, -1)$.

5. A plane has equation $z = 5x - 2y + 7$.

(a) Find a value of a making the vector $a\mathbf{i} + \mathbf{j} + 0.5\mathbf{k}$ normal (i.e., perpendicular) to the plane.

(b) Find a value of b so that the point $(b + 1, b, b - 1)$ lies in the plane.

6. Consider the plane $x + 3y - 2z = 4$ and the vector $\mathbf{v} = \langle 1, 2, 3 \rangle$.

(a) Find a normal vector to the plane.

(b) What is the angle between $\mathbf{v}$ and the vector you found in part (a)?

(c) What is the angle between $\mathbf{v}$ and the plane?

7. *Spicy:* Find the shortest distance between the planes $2x - 5y + z = 10$ and $z = 5y - 2x$.

8. Give an example of a vector $\mathbf{v}$ whose cross product with $\mathbf{u} = \mathbf{i} + \mathbf{j}$ is parallel to $\mathbf{k}$.

9. Let $f(x, y) = 3x^2 + 4y^2 - axy$.

(a) Find the values of a such that the graph of f slopes upwards when moving from the point $(1, 2)$ in the positive x-direction.

(b) Find the values of a such that the graph of f slopes upwards when moving from the point $(1, 2)$ in the positive y-direction.

10. Give an example of a nonlinear function $f(x, y)$ such that $f_x(0, 0) = 3$ and $f_y(0, 0) = 4$.

11. Let $h(x, y)$ be a differentiable function such that $h(300, 200) = 50$, $h_x(300, 200) = 8$, and $h_y(300, 200) = -6$. Estimate the value of $h(305, 196)$.

12. If $z = f(x) + yg(x)$, what can you say about z_{yy}? Explain!

13. Let $f(x, y) = x^2 + 2y^2$. Find the directional derivative at the point $(0, \pi/4)$ in the direction of $\mathbf{i} + \mathbf{j}$.

14. Let $g(x, y) = ye^{-x^2} + 2y$. Find the gradient of g and the maximum value of the directional derivative of g at the point $(0, 4)$.