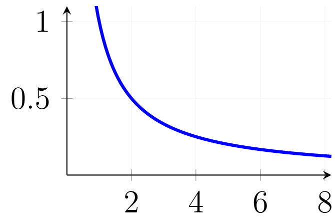
Series of Real Numbers

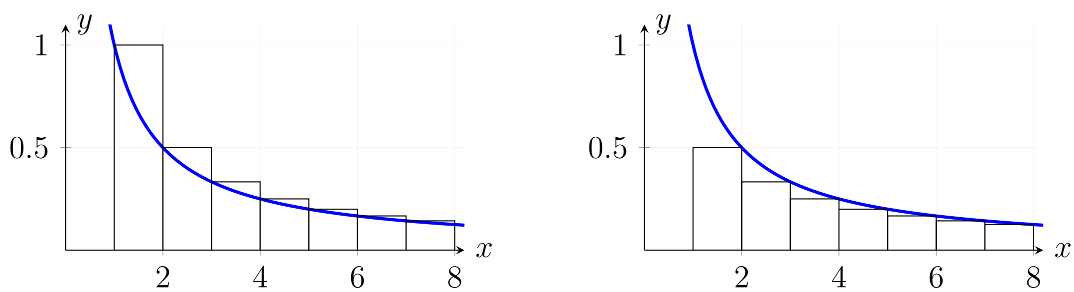
1. **Lana:** Hey, Simon! Want to draw with me?

   Simon: Sure, but I really miss integrals. Can we please go back? I'll draw $f(x) = \dfrac{1}{x}$.

   Lana: YAY! This reminds me of when we did improper integrals.

   Simon: I'll draw the left and right Riemann sum rectangles to approximate $\displaystyle\int_1^\infty \frac{1}{x}\,dx$.

   Lana: Now we just have to add up the areas of all of the rectangles!

   Simon: Um....I thought I told you I wanted to go back to integrals...

   Group Task: Write the left and right rectangle areas as sums that goes on forever. ☞ They are slightly different!

   Simon: We know $\displaystyle\int_1^\infty \frac{1}{x}\,dx$ diverges, so the area underneath $y = \dfrac{1}{x}$ is infinite.

   Lana: I guess that this must mean that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ *also* diverges!

   Group chat: Why did Lana say this?

   Simon: I think we can use the same reasoning to argue that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ *converges.* Amazing!

   Group chat: Why does Simon think that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges?

   Group chat: For which value(s) of p do you think $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges? ☞ Take a wild guess. I bet you're right!

2. (a) Is the function $f(x) = \dfrac{2x}{(x^2+4)^2}$ continuous as $x \to \infty$?

(b) Is the function $f(x) = \dfrac{2x}{(x^2+4)^2}$ (at least eventually) positive as $x \to \infty$?

(c) Is the function $f(x) = \dfrac{2x}{(x^2+4)^2}$ (at least eventually) decreasing as $x \to \infty$?

☞ IF the answers to (a), (b), and (c) are all YES, you may TRY using the integral test.

(d) Does $\displaystyle\int_1^\infty \frac{2x}{(x^2+4)^2}\,dx$ converge or diverge?

(e) Does $\displaystyle\sum_{n=1}^\infty \frac{2n}{(n^2+4)^2}$ converge or diverge?

3. (a) If $n \geq 1$: $\dfrac{1}{n^2+n}$ is bigger/smaller than $\dfrac{1}{n^2}$.

☞ circle one

(b) The sum $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+n}$ is bigger/smaller than the sum $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2}$.

(c) **Ava:** I really want to know if $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+n}$ converges or diverges.

Milo: That series totally converges. Look at part (b) above!

Group Chat: Why do you think Milo is arguing that $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+n}$ converges?

4. (a) **Milo:** You can also figure out the answer to $\displaystyle\sum_{n=2}^\infty \frac{1}{n-\sqrt{n}}$ this way.

Group chat: What is Milo thinking? Does that series converge or diverge?

☞ Do you see why this series starts at $n = 2$?

(b) **Ava:** But Milo, your method is really picky! It *fails* to work for $\displaystyle\sum_{n=2}^\infty \frac{1}{n^2-1}$.

Milo: I guess so, but no worries! We know $\dfrac{1}{n^2-1} \approx \dfrac{1}{n^2}$, so it works out OK.

Group Chat: What are Milo and Ava talking about?