Geometric Series, Convergence, and Divergence

1. Here is a geometric series: $a + ar + ar^2 + ar^3 + \cdots = \sum_{n=0}^{\infty} ar^n$

Consider a partial sum of this series:

$$s_n = a + ar + ar^2 + ar^3 + \cdots + ar^n$$

(a) What is $r \cdot s_n$?

$r \cdot s_n =$

(b) You might notice that s_n and $r \cdot s_n$ are VERY similar. What happens when you subtract them?

$s_n - r \cdot s_n =$

(c) Use the above to solve for s_n.

☞ Look, you found a formula for s_n. WOW!

(d) In order for the geometric series to converge, you need $\lim_{n\to\infty} s_n$ to be a number (not infinity or oscillating). Try a few specific values for r. For what values of r does the limit exist?

☞ Try positive, negative, big, small, etc.

(e) If r is such that the geometric series converges, what does it converge TO?

2. Determine if the series converges or diverges. If the series converges, find the value that it converges to.

(a) $\sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n$

☞ Hint: Identify a and r.

(b) $\sum_{n=1}^{\infty} 4\left(\frac{1}{3}\right)^n$

(c) $\sum_{n=1}^{\infty} \frac{e^{n+1}}{2^n}$

3. Write an example of a geometric series that converges to 5. See if you can write an example *different* from everyone else at your table.

☞ The SUM of your series should be 5.

4. Which of the following series diverge? How do you know?

☞ Think about what happens to the partial sums as you add more and more terms.

$$\sum_{n=0}^{\infty}(-2)^n =$$

$$\sum_{n=1}^{\infty} n =$$

$$\sum_{n=2}^{\infty} \frac{n+1}{n} =$$

$$\sum_{n=1}^{\infty} \frac{n+1}{3n} =$$

5. **Erez:** I really like this series, but I don't know if it converges.

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{32}+\cdots$$

Group Chat: Help Erez out. Does the series converge?

☞ Hint: Do not find all the partial sums. Focus on s_1, s_2, s_4, s_8, s_{16}, etc.

Ava: Since you figured out Erez's series, can you tell me whether my series converges or diverges?

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\frac{1}{15}+\frac{1}{16}+\frac{1}{17}+\cdots$$

Group chat: How can you apply what you learned about Erez's series to answer Ava's question?