# Sequences and Limits

**1.** Find a formula for the $n$th term of the following infinite sequences:

Go ahead and assume we start with $n = 1$. It's not always possible to find a formula, but for these examples you can.

$$\{a_n\} = \left\{e \cdot \pi, \tfrac{e\cdot\pi^2}{2}, \tfrac{e\cdot\pi^3}{3}, \tfrac{e\cdot\pi^4}{4}, \tfrac{e\cdot\pi^5}{5}, \ldots\right\}$$

$$\{b_n\} = \left\{1, \tfrac{2}{3}, \tfrac{3}{5}, \tfrac{4}{7}, \tfrac{5}{9}, \tfrac{6}{11}, \ldots\right\}$$

$$\{c_n\} = \left\{1, \tfrac{1}{2}, \tfrac{1}{6}, \tfrac{1}{24}, \tfrac{1}{120}, \tfrac{1}{720}, \ldots\right\}$$

$$\{d_n\} = \{-1, 1, -1, 1, -1, 1, -1, 1, -1, \ldots\}$$

$$\{j_n\} = \left\{-1, \tfrac{1}{2}, \tfrac{-1}{6}, \tfrac{1}{24}, \tfrac{-1}{120}, \tfrac{1}{720}, \ldots\right\}$$

$$\{k_n\} = \{3, 7, 3, 7, 3, 7, \ldots\}$$

**2.** For the sequences above...

(a) What is $\lim_{n\to\infty} a_n$?

(b) What is $\lim_{n\to\infty} b_n$?

(c) What is $\lim_{n\to\infty} c_n$?

(d) What is $\lim_{n\to\infty} d_n$?

(e) What is $\lim_{n\to\infty} j_n$?

(f) What is $\lim_{n\to\infty} k_n$?

**3.** **Erez:** This isn't calculus! I miss integrals. Improper integrals were pretty interesting.

**Chloe:** Agreed. But did you know there are similarities at least? We still get to say the words "converge" and "diverge."

**Erez:** We have not defined those words in the context of sequences!

**Chloe:** Oh, but we can! The definition is not really any different!

**Group chat:** What does Chloe mean? How would you define converge and diverge in the context of sequences?

**4.** Look again at the sequences at the top of this page. Which sequences converge? Which diverge?

**5.** For each of the following formulas for $a_n$, does the sequence $\{a_n\}$ converge or diverge? If it converges, what does it converge to?

(a) $a_n = \dfrac{27}{n^2 - 2}$

(b) $a_n = \dfrac{n^3 + 4n^2 + 1}{3n^3 - 2n + 4}$

(c) $a_n = \dfrac{n^3 + 4n^2 + 1}{3n^4 - 2n + 4}$

(d) $a_n = \sin(\pi n)$ Careful! ☺

(e) $a_n = \cos(\pi n)$

(f) $a_n = \dfrac{n^7}{2^n}$

**6. Phil:** Hey there, Renita! Have I shown you my favorite sequence? Here it is:

$$2.9, 2.95, 2.959, 2.9595, 2.95959, 2.959595, \ldots$$

**Renita:** Oh, I get it! I love patterns.

**Phil:** The problem is I am having trouble finding a formula for it, so I cannot figure out how to argue that this sequence converges, even though it clearly does!

**Renita:** The terms of your sequence are getting larger and larger, right?

**Phil:** You betcha!

**Renita:** And none of the terms of your sequence are larger than 3, right?

**Phil:** Yeah...

**Renita:** That's enough information to guarantee your sequence converges!

**Group chat:** Why does Renita say "that's enough information to guarantee your sequence converges"?

**7.** Try to modify Renita's argument for the sequence

$$2.1, 2.01, 2.001, 2.0001, 2.00001, 2.000001, 2.0000001, \ldots$$